Glider Report
Phoenix: Raising Grades from the Ashes

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I. Introduction

1.1 The Basic Equations of Flight Mechanics

What needs to be known to understand how a glider flies? This discussion begins with understanding the environment that the glider is flying in. For this, there are three values to consider: temperature (T), pressure (p), and density (ρ). In theoretical calculations, where standard atmospheric conditions are assumed, temperature is 288 K, pressure is $1.01\times 10^{5}\frac{N}{m^{2}}$, and density is 1.23 $\frac{kg}{m^{3}}$. On flight day, a mobile weather app was used to collect temperature and pressure; however, density was not provided. Atmospheric density can be calculated using the ideal gas law equation,

$$\rho = \frac{p}{RT} \tag{1}$$

where R is the gas constant for air with the value of 287 $\frac{J}{kg\bullet K}$.

Atmospheric density, ρ, is used in calculating the Reynolds number (*Re*), another important value in flight mechanics, through

$$Re = \frac{\rho ul}{\mu} \tag{2}$$

where u is flight speed (m/s), l is length (m), and μ is dynamic viscosity ($\frac{N\cdot s}{m^{2}}$). This equation (Eq. 2) can be rewritten in terms of kinematic viscosity, ν, where

$$\nu = \frac{\mu}{\rho} \tag{3}$$

in m^2/s, and the new, rewritten, equation for Re is

$$Re = \frac{ul}{\nu}. \tag{4}$$

ρ, from equation 1, is also used to calculate dynamic pressure, q, in the equation

$$q = \frac{1}{2}\rho u^{2} \tag{5}$$

with units of $\frac{N}{m^{2}}$ where u is, once again, speed in m/s.

Dynamic pressure, q, is used in calculating aerodynamic forces such as lift and drag. The lift equation is given as

$$L = qSC_L = \tfrac{1}{2}\rho u^{2} SC_{L} \tag{6}$$

with S being the planar surface area, and C_L as the lift coefficient. This equation can also be rewritten to solve for C_L. Because the value of gamma is so small, the simplifying assumption is made that lift is approximately equal to weight, expressed as

$$L = W \quad \textbf{(7)}$$

where W is calculated by multiplying the mass of the glider (m) by the gravitational acceleration (g = 9.8 m/s^2).

The base equation for parasite drag,

$$D = qSC_{D,0}, \quad \textbf{(8)}$$

where $C_{D,0}$ is the drag coefficient, is used in calculating the components that make up the total drag calculation—

$$D_{tot} = D_0 + D_i = (D_f + D_{pro}) + D_i \quad \textbf{(9)}$$

where D_i represents induced drag, and D_0 is composed of profile drag (D_{pro}) and friction drag (D_f). The values used for S and C_D vary with each component of drag. For profile drag,

$$D_{pro} = C_{d,min} qS \quad \textbf{(10)}$$

where S is the planar surface area of the wing and $C_{d,min}$ is the minimum coefficient of drag. Friction drag is the sum of the friction drag components from the fuselage ($_{fus}$), horizontal stabilizer ($_{hs}$), and vertical stabilizer ($_{vs}$)—

$$D_f = q(C_{f,\,fus}S_{W,\,fus} + C_{f,\,hs}S_{W,\,hs} + C_{f,\,vs}S_{W,\,vs}) \quad \textbf{(11)}$$

where C_f is the coefficient of friction ($C_{f,\,fus}$, for example, indicates the coefficient of friction for the fuselage), calculated by

$$C_{f,\,laminar} = \frac{1.328}{\sqrt{Re_L}} \quad \textbf{(12)}$$

for laminar flows (where Re is less than 5×10^{-5}) and

$$C_{f,\,turbulent} = \frac{0.074}{\sqrt[5]{Re}} \quad \textbf{(13)}$$

for turbulent flows, and S_W stands for the wetted surface area. The final component of total drag, induced drag, is calculated by

$$D_i = qSKC_L^2 \quad \textbf{(14)}$$

where S is the wetted surface area of the wing, C_L is the coefficient of lift, and

$$K = \frac{1}{\pi e_v AR} \quad \textbf{(15)}$$

with AR representing the aspect ratio of the glider, calculated as

$$AR = \frac{b}{c} \quad \textbf{(16)}$$

where b is wing span length, c is chord length, and e_v is Oswald Efficiency, calculated by

$$e_v = \frac{1}{1.05 + 0.007\pi AR}, \quad \textbf{(17)}$$

with AR being the aspect ratio, calculated using Eq. 16 (*Reference 2)*.

A typical aircraft in steady, level flight will have a force diagram that looks like Figure 1 below.

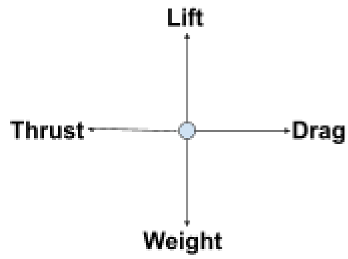

Figure 1. Typical Aerodynamic Force Diagram

Gliders, however, do not have a thrust mechanism. As such, their force diagram differs, as shown below in Figure 2,

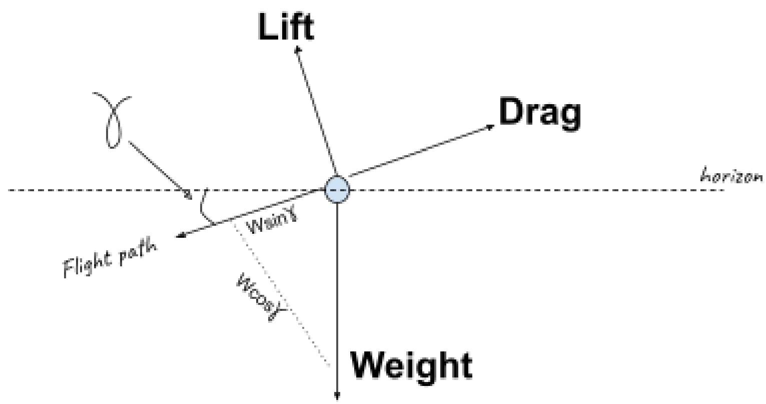

Figure 2. Glider Aerodynamic Force Diagram

where γ is the glide angle caused as a result of the lack of thrust. For aircrafts following the force diagram in Figure 1, the ratio

$$\frac{T}{W} = \frac{D}{L} \tag{18}$$

applies. This ratio's values are altered for gliders such that

$$\frac{W sin(\gamma)}{W cos(\gamma)} = \frac{D}{L},$$

which is simplified to

$$tan(\gamma) = \frac{1}{L/D}. \tag{19}$$

From this, it is evident that γ_{min} occurs at $(\frac{L}{D})_{max}$. Decreasing γ results in a flatter slope of the glider's path, which, when minimized, results in γ_{min}, represents the optimum gliding angle. To achieve γ_{min}, a specific speed, U_γ, is necessary. This calculation begins with using the equation for speed,

$$U = \sqrt{\frac{2W}{\rho SC_L}}, \tag{20}$$

written by rearranging Eq. 6 and Eq. 7.

Flight efficiency in aerodynamics is denoted as $\frac{L}{D}$. As such, at $(\frac{L}{D})_{max}$, the craft is also at maximum efficiency, which results in the craft reaching its maximum range for the flight.

Another factor of maximum range is minimum drag which occurs when the derivative of drag with respect to q is equal to zero, or

$$\frac{d}{dq}\left[qSC_{D,0} - \frac{KW^2}{qS}\right] = 0,$$

$$SC_{D,0} - \frac{KW^2}{q^2S} = 0$$

$$C_{D,0} = \frac{KW^2}{q^2S^2} = KC_L^2$$

$$C_{D,0} = C_{D,i} \tag{21}$$

where $C_{D,0}$ is the coefficient of parasite drag, and $C_{D,i}$ is the coefficient of induced drag, defined as

$$C_{D,i} = KC_L^2. \tag{22}$$

which can be substituted into Eq. 21 to show that

$$C_{D,0} = KC_L^2 \tag{23}$$

and, therefore,

$$C_L = \sqrt{\frac{C_{D,0}}{K}}. \tag{24}$$

Given the components of $(\frac{L}{D})_{max}$, and returning to the base equation for speed (Eq. 20), (Eq. 24) can be substituted in for C_L to provide the equation for U_γ—

$$U_\gamma^2 = \frac{2W}{\rho S}\sqrt{\frac{K}{C_{D,0}}}. \tag{25}$$

1.2 Predictions and Measurable Quantities

What quantities are measured in order to analyze the glider's performance for the test flight? The quantity most telling of performance would be the distance that the glider flies, also referred to as the range. To find the range of the glider, the equation

$$R = h(\frac{L}{D})_{max} \tag{26}$$

is used with h representing the takeoff height of the glider, in meters.

Original predictions for glider performance were based on a mass of 0.6kg, a takeoff height of 2m, and a standard atmospheric density of 1.23kg/m^3; however, the mass was measured to be

0.85kg on flight day, with a 1.75m launch height, and atmospheric density was calculated as 1.24kg/m^3. Given this, predictions of maximum range for both masses are calculated.

Beginning with original predictions, $(L/D)_{max}$ is found by graphing (L/D), made from Eq. 6 and Eq. 9, against U

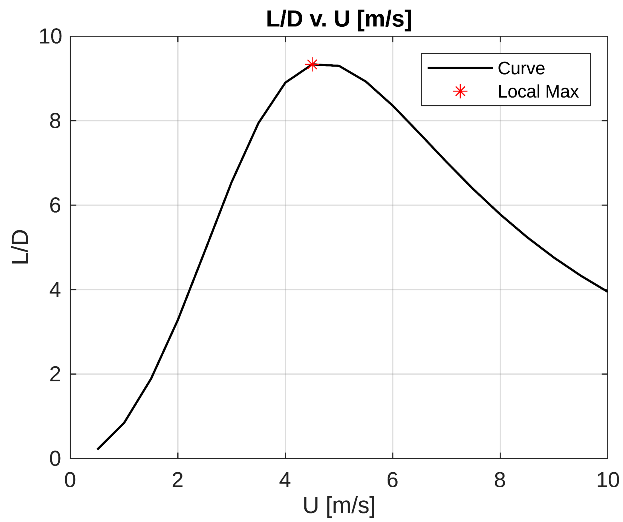

Figure 3. (L/D) v. U (m/s) at 0.6kg

from which the resulting value is 9.33. Using Eq. 26 and a launch height of 2 meters, Range is

$$R = (2)(9.33) = 18.7 \text{ meters}$$

For the actual mass on flight day with an atmospheric density of 1.24kg/m^3, L/D is graphed again to produce

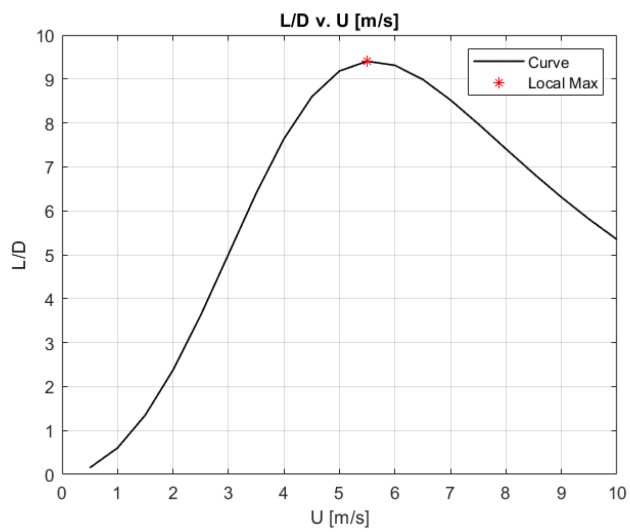

Figure 4. (L/D) v. U (m/s) at 0.85kg

with a maximum value of 9.4, and, therefore, a maximum range of 16.45 meters.

The experimental value for maximum range will be measured and compared later in the report.

II. Materials and Methods

2.1 Glider

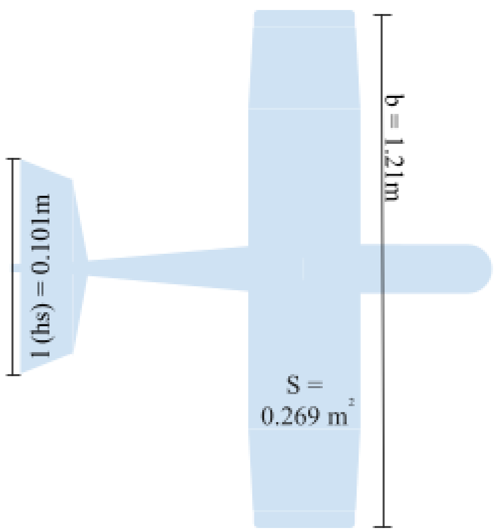

Figure 5. SNACKS Glider Plan View

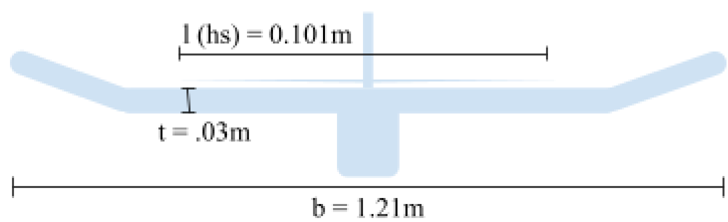

Figure 6. SNACKS Glider Front View

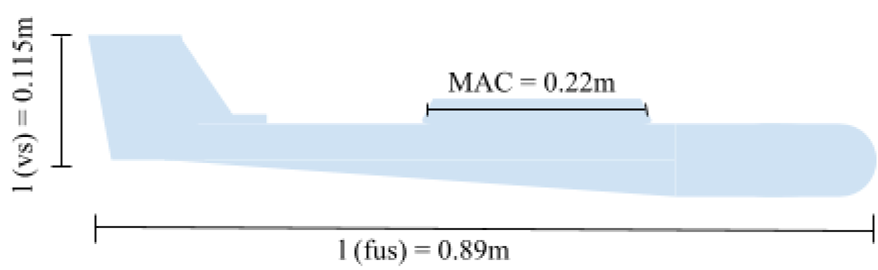

Figure 7. SNACKS Glider Side View

Table 1. Glider Profile Measurements

Measurement	Value	Unit	Value	Unit
Tip-to-tip wing span *(b)*	121	cm	1.21	m
Wing Planform Area *(S)* *= bc = (121)(21.5)*	2600	cm^2	0.269	m^2
Length of Fuselage *(l (fus)*)	89	cm	0.89	m

Length of Horizontal Stabilizer (*l (hs)*)	10.1	cm	0.101	m
Length of Vertical Stabilizer (*l (vs)*)	11.5	cm	0.115	m
Mean Aerodynamic Chord (*MAC or* $\bar{c}$) $= (C_{tip} + C_{root}) / 2 = (19.4+23.5)/2$	21.5	cm	0.215	m
Aspect Ratio (*AR*) $= b/c = 121/21.5$	5.63	–	–	–
Maximum Thickness of airfoil = thickness (*t*)/c = 3.18cm/21.5cm = 0.15	15	% (of c)	–	–
Maximum Camber of Airfoil (as a percent of the chord) = camber/chord = 1.43cm/22.9cm = 0.062 (6 if rounded to 1 digit)	6.2	% (of c)	–	–
Distance from leading edge to location of max camber 8.26cm /22.9cm = 0.361 = 36.1% → 40%	40	%(10s of % of c)	–	–

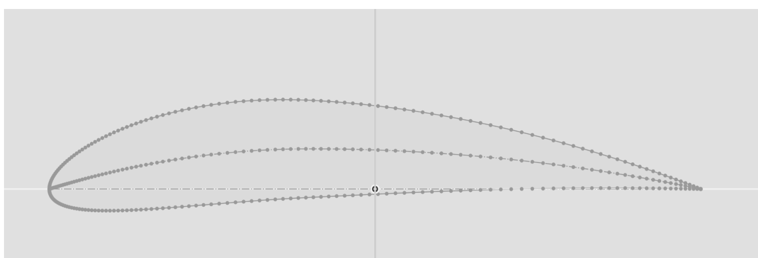

Figure 8. NACA 6415 Section Profile [5]

Equivalent NACA xyzz geometry: NACA 6415

x = max camber as a percent of the chord = 6% = 6

y = distance from leading edge to max camber as 10s percent of chord = 40% = 4

zz = max thickness as a percentage of the chord = 15% = 15

2.2 Testing Procedures

The quantities measured during the flight test can be organized into two categories: measured quantities (Table 2), where the values were recorded after each test flight, and constant measurements (Table 3), where the values did not change between each test flight.

The measured quantities include mass, speed, flight duration, distance in the x-direction (X), and distance in the y-direction (Y).

Mass (kg) - The mass of the glider was measured before each flight test, as adjustments were made to the left side of the wing. Clay was added to straighten the glider's flight path, however not significantly enough to change the measured kilograms. Additionally, a rudder was added to the glider's horizontal stabilizer after flight test two.

Speed U_o(m/s) - The initial speed of the glider was measured by the glider launcher.

Flight Duration (s) - The measured flight time was measured by videography, as a group member filmed the procession of each launch. The time started once the glider's nose reached the end of the launcher and concluded when any part of the glider impacted the ground.

Distance in X Direction (m) - Distance was measured in the vertical direction from the launcher, or the length of the glide path.

Distance in Y Direction (m) - Distance was measured in the horizontal distance from the launcher. Positive distance was measured on the right side of the vertical tape measurer when facing the field from the launcher, while negative distance was measured on the left side.

Table 2. Flight Test Measured Quantities

Flight Number	U_o (m/s)	Flight Duration (s)	X (m)	Y (m)	Mass (kg)
1	6.3	1.5	8.4	1.6	0.85
2	7.7	2.5	16.6	4.6	0.85
3	7.8	2.2	14.6	2.7	0.85
4	7.1	2.0	12.3	2.4	0.85
5	9.1	4.1	26.5	0.15	0.85
6	5.3	1.5	7.1	-0.15	0.85
7	9.2	4.3	29.4	1.5	0.85

The constant quantities include platform height, temperature, pressure, and density.

Platform Height (m) - Measured by Glider Launcher Operators on Glider Day.

Temperature (K) - Recorded from The Weather Channel at 6:46 am.

Pressure (N/m^2) - Recorded from The Weather Channel at 6:46 am.

Density (kg/m^3) - Calculated with the Ideal Gas Law (Eq. 1) using the constant value R, and the measured temperature and pressures.

Table 3. Constant Quantities

Platform Height (m)	1.75
Temperature (K)	280
Pressure (N/m^2)	1.0 x 10^5
Density (kg/m^3)	1.24

III. Results

3.1 Reynolds Numbers and Drag Estimates

The values in Table 4 were calculated using (Eq. 4), (Eq. 5), (Eq. 11), and (Eq. 14), the data from Table 2, and the values for the viscosity of air, $\boldsymbol{\nu}$ = 13.88x$10^{-6}\ m^2/s$, found using *Reference 6* and density, $\rho = 1.24\ kg/m^3$, referenced in (Sec. 2.2).

Table 4. Reynolds Number and Frictions

Launch #	Re	D_f	C_f	U (m/s)
1	9.76×10^4	7.4×10^{-3}	.182	6.3
2	1.19×10^5	7.15×10^{-3}	.263	7.7
3	1.21×10^5	7.12×10^{-3}	.269	7.8
4	1.41×10^5	7.26×10^{-3}	.227	7.1
5	1.3×10^5	6.91×10^{-3}	.355	9.1
6	8.21×10^4	7.7×10^{-3}	.134	5.3
7	1.43×10^5	6.89×10^{-3}	.362	9.2

3.2 Range vs. Speed— Theory and Tests

(Fig. 9) is a visual graphical representation of the glider's flight path during each flight round with lateral distance (x) and horizontal distance (y). The legend to the right of the graph lists the landing points of each flight starting with flight 1 at the top and ending with flight 7.

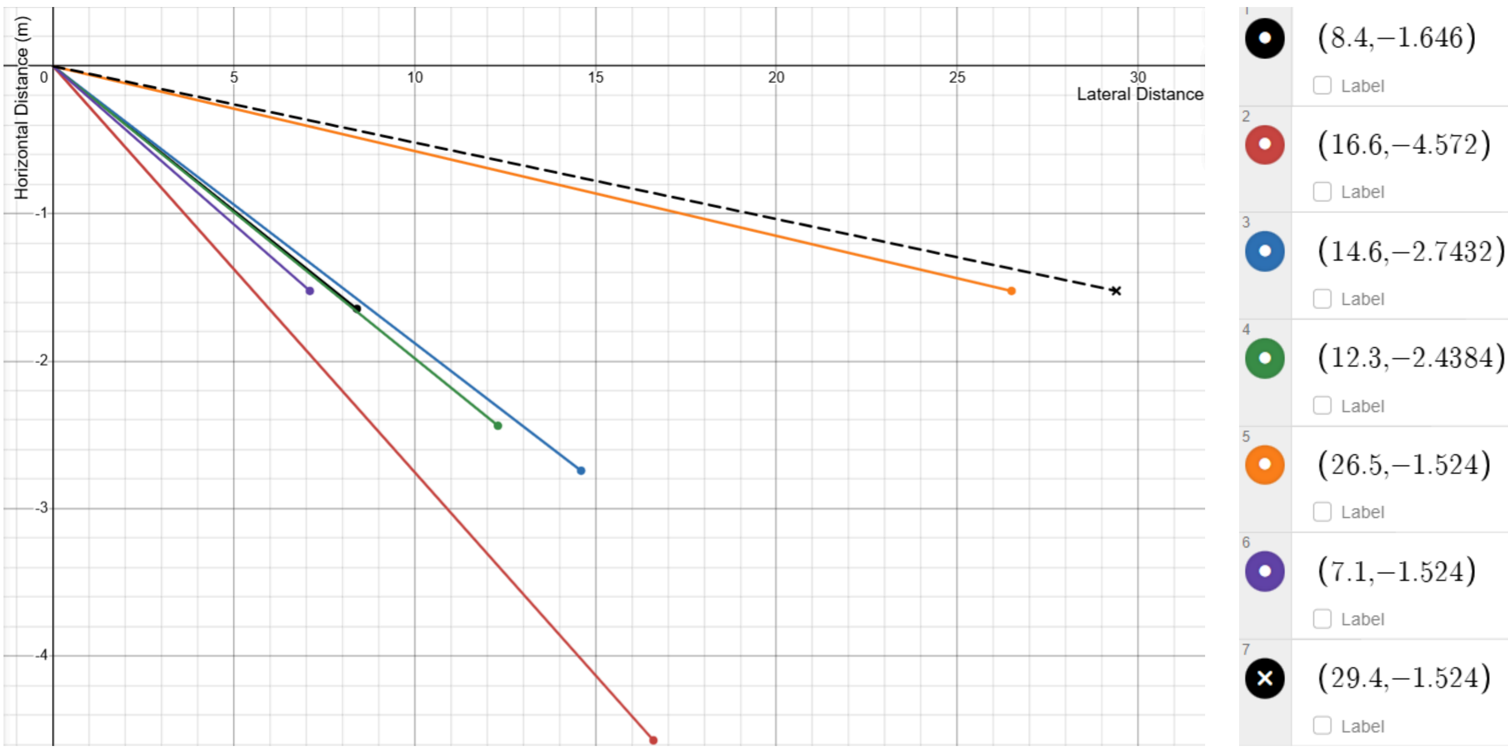

Figure 9. Experimental Range Data and Landing Points in meters

Figure 10. Theoretical Glider Range Data with experiments in order Graphed using Eq. 26

Table 5. Theoretical and Experimental Ranges and Differences

Flight #	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Speed (m/s)	6.3	7.7	7.8	7.1	9.1	5.3	9.2
Theoretical Range (m)	22.4	24.0	24.0	23.8	22.6	18.8	22.4
Experimental Range (m)	8.56	17.2	14.9	12.6	26.6	7.28	29.5
Difference (m)	13.8	6.77	9.10	11.2	3.99	11.5	7.06

Mean Experimental Range: 16.7m
Mean Theoretical Range: 22.6m
Mean Difference in Ranges: 9.06m

Experimental Range Standard Deviation: 7.98m
Theoretical Range Standard Deviation: 1.49m
Difference in Ranges Standard Deviation: 3.1m

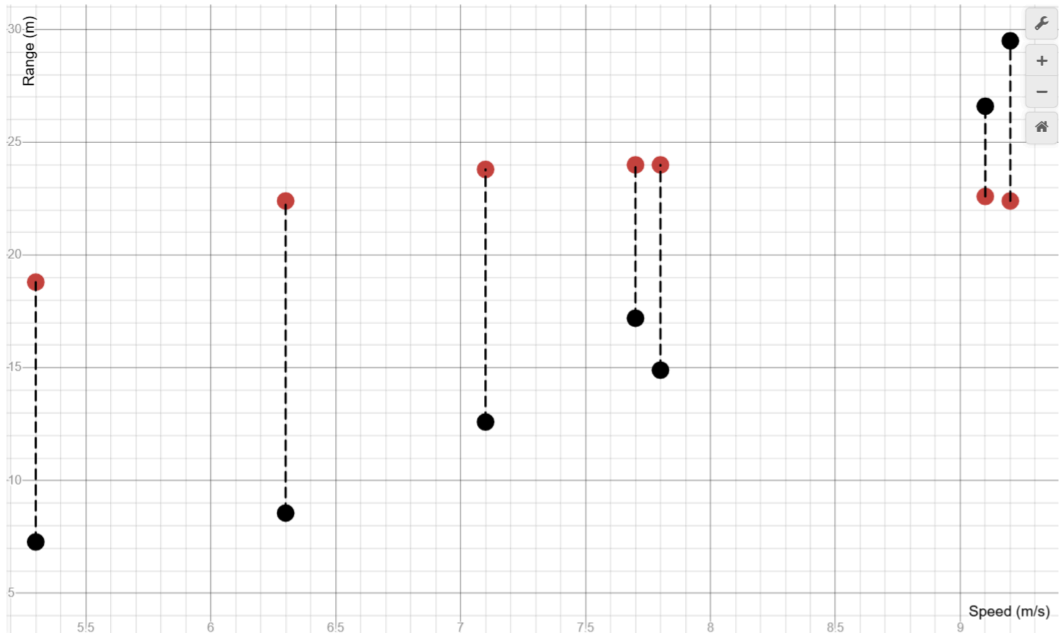

Figure 11. Theoretical ranges (Red) vs Experimented Ranges (Black) at tested speeds

The experimental data found does not accurately reflect the expected ranges gained from theoretical calculations. The graph predicted that the glider would find its maximum range at a launch speed of U = ~7.6 meters per second. In the experimented data, however, the range continued to trend upward as the glider reached higher speeds, as shown by the black dots in (Fig. 11), and reached a maximum range at 9.2m/s. The ranges for the experimented data were below the theoretical values at low and mid speeds, but above at high speeds. As shown by the theoretical ranges standard deviation (1.49m) compared to the experimented ranges standard deviation (7.98m) the experimental data had more than 5 times the variation in ranges than the theoretical values. By comparing the mean of Theoretical Ranges (22.6m) to the mean of the Experimented ranges (16.7) it is evident that overall the glider was not able to travel as far as predicted. [1]

[1] Following the fourth glider launch, it was noted that a Launcher Operator was assisting the glider by holding up the left side of the wing. This is believed to have caused the glider to veer to the right, possibly affecting the max range around the 7.6 m/s speed. After the assistance to the wing ceased, the glider exhibited a straighter flight path as noted in the fifth launch (Table 2).

IV. Discussion

4.1 Comparison of Theory and Experiment

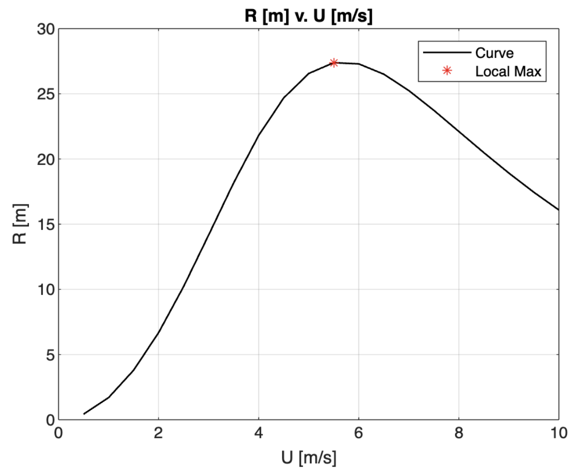

Figure 12. R v. U [m/s]; (R_{max} = 27.4m at U = 5.50m/s) [m = 0.60kg and h = 2.00m]

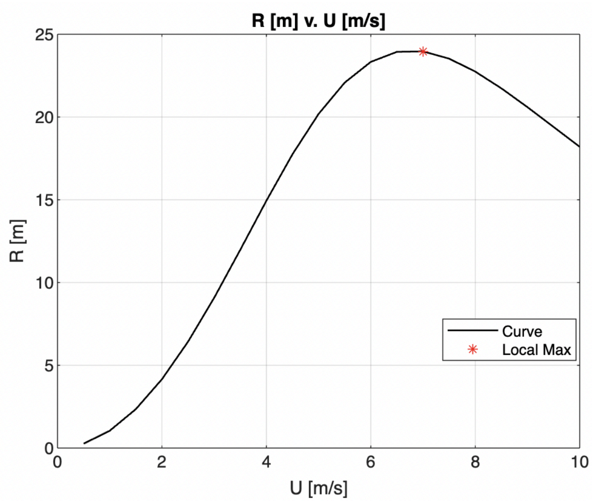

Figure 13. R v. U [m/s]; (R_{max} = 23.9m at U = 7.00m/s) [m = 0.85kg and h = 1.75m]

The theoretical value of the glider's mass was 0.6 kilograms with a takeoff height of 2m. With this mass, the maximum range of 27.4m would be achieved when the flight speed is 5.50m/s (Fig. 12). However, the actual mass value was 0.85 kilograms with a takeoff height of 1.75m. With the actual mass and takeoff height, the maximum range of 23.9m is lower and achieved at a higher flight speed of 7m/s (Fig. 13).
Lift is proportional to weight, so the increase in the glider's weight also increases its lift (Eq. 7). The decrease in the takeoff height would negatively affect the range (Eq. 26).

However, the takeoff height only decreased by 12.5%, while the weight, and therefore lift, increased by 26.4%. Therefore, the glider should travel further despite the decrease in height.

To compare tests, the flight speed for test 6 was 5.3 m/s and the glider traveled 7.1 m in the x-direction, while test 5 was conducted at a higher speed of 9.1 m/s and the glider traveled 26.5 m in the x-direction (Table 5).

This is consistent with the theoretical data which shows the increase in weight from the predicted value the glider will reach its maximum range at higher speeds. During test 6, the glider should have traveled further according to the theoretical values of weight and takeoff height (Fig. 12), however, it did not. It did, however, travel further with a higher speed.

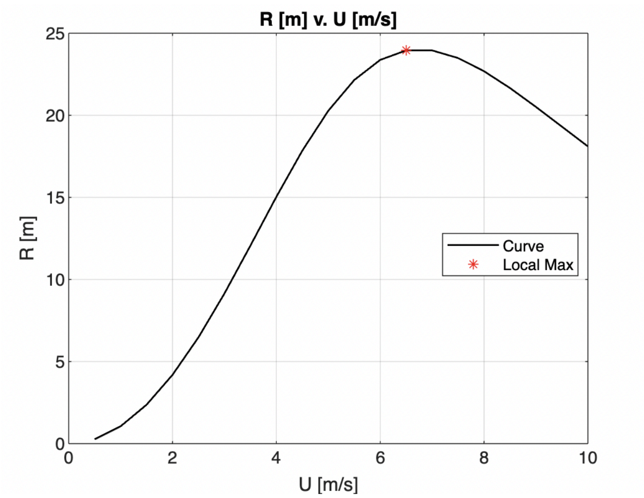

Figure 14. R (m) v. U (m/s); (R_{max} = 23.9m at U = 6.5m/s) [ρ = 1.24kg/m³]

Theoretically, the atmospheric density is 1.23kg/m³, but on test day, the reported value was 1.24kg/m³. With this difference, the glider should have reached the maximum range of 23.9m at a flight speed of 6.5m/s (Fig. 14). The maximum range did not change from the previous theoretical example, but the flight speed it was achieved at did (Fig. 13). This is because at a higher density, the glider did not need to travel at a higher speed to generate more lift as the higher density provided more lift.

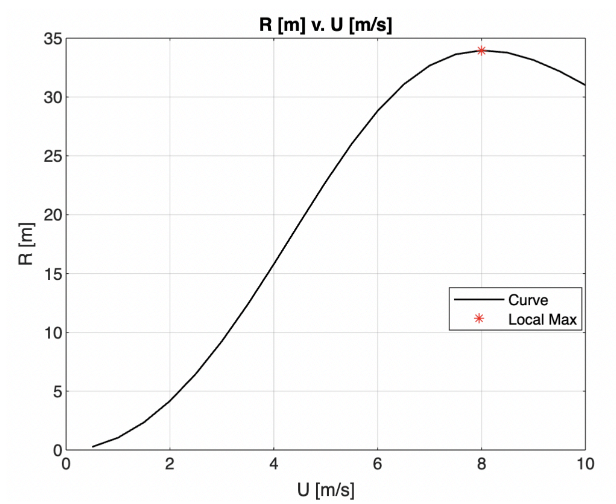

Figure 15. R (m) v. U (m/s); (R_{max} = 33.9m at U = 8m/s) [$C_{D,O}$ = 0.01]

Induced drag on the glider could have increased due to the increase in surface roughness from the addition of tape. With the actual mass of 0.85kg and the theoretical drag coefficient's value of 0.02, the maximum range of 23.9m occurred at a speed of 7.00m/s (Fig. 13). However, the actual data showed that the glider traveled furthest in the x-direction when the speed was 9.1m/s. This leads to the conclusion that the drag coefficient's value is lower than previously predicted.
As the drag coefficient decreases, the parasite drag decreases as well (Eq. 8). This decreases the total drag of the glider (Eq. 9). If the total drag decreases, then the range will increase (Eq. 26).

Decreasing the value of the drag coefficient to 0.01 to test this theory shows that the maximum range was 33.9m and occurred when the speed was 8m/s (Fig. 15). This is consistent with the flight test data. If the drag coefficient's value was 0.01, it would make sense that at a speed of around 9m/s, the range found was lower than the maximum, but still close to that number. It would also account for the added tape, which would increase the drag coefficient's value, but with an already low value, the tape didn't make a huge difference in the data.

Overall, the glider's performance did vary with speed as expected based on the updated values of the drag coefficient, mass, and takeoff height.

4.2 Wing Loading

Table 6. Wing Loading and Weight Specs for Various Flying Devices

Flying Object	Type	Wing Loading (kg/m^2)	Weight (kg)
[1] Blue Underwing	Butterfly	0.444	0.0012
[1] Laughing Gull	Bird	2.04	0.336
[1] Common Tern	Bird	2.34	0.117
[4] Ozone Buzz Z3 MS	Paraglider	3	77.4
Wooden Glider	Glider	3.16	0.85
[4] Ozone Buzz Z3 MS	Paraglider	3.5	90.3
[4] General	Hang glider	6.80	102
[4] Wills Wing Sport 2 155	Hang Glider	6.6	94.8
[4] Gin Fluid 11	Speed Flyer	12.7	140

[4] General	Microlift Glider	18	220
[1] Boeing 747-400	Plane	769	403000

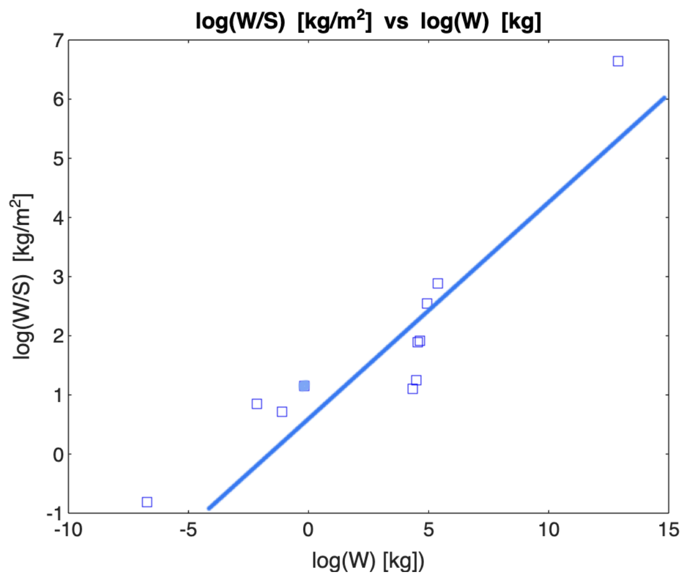

Figure 16. log(W/S) (kg/m^2) v. log(W) (kg) [Glider Value Highlighted]

The wing loading for the glider is not unusual. The value of the logarithmic of the glider's wing loading and weight falls within a reasonable range from the line of best fit (Fig. 16). This is expected as the glider was able to maintain lift for long ranges at a flight speed that was reasonable according to Figures 13, 14 and 15. If the glider had an unusual wing loading, it would have needed to go faster or slower than the predicted values to generate the correct amount of lift to achieve the maximum range.

4.3 (L/D)$_{max}$

To find an equation for $\left(\frac{L}{D}\right)_{max}$ using only K and $C_{D,0}$, first it is necessary to find the expression for the coefficient of total drag,

$$C_D = C_{D,0} + KC_L^2 \quad \textbf{(27)}$$

where $C_{D,0}$ is parasite drag and KC_L^2 represents induced drag. Then find the expression for $\left(\frac{L}{D}\right)$,

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D,0}+KC_L^2} \quad \textbf{(28)}$$

in which lift, L, and drag, D, are put in terms of coefficient of lift, C_L, coefficient of drag, C_D, and K. Then find the derivative of the equation to determine $\left(\frac{L}{D}\right)_{max}$,

$$\frac{d}{dC_L}\left(\frac{C_L}{C_{D,0}+KC_L^2}\right) = \frac{(C_{D,0}+KC_L^2)-C_L(2KC_L)}{(C_{D,0}+KC_L^2)^2} = \frac{C_{D,0}-KC_L^2}{(C_{D,0}+KC_L^2)^2} \quad \textbf{(29)}$$

which is then simplified into the following expression,

$$C_L^2 = \frac{C_{D,0}}{K}. \quad \textbf{(30)}$$

Now substitute these values to find a new definition of the coefficient of total drag,

$$C_D = C_{D,0} + KC_L^2 = C_{D,0} + K\left(\frac{C_{D,0}}{K}\right) = 2C_{D,0} \quad \textbf{(31)}$$

and using this new definition of the coefficient of total drag, an equation for $\left(\frac{L}{D}\right)_{max}$ in terms of K and $C_{D,0}$ can be found

$$\left(\frac{L}{D}\right)_{max} = \frac{\sqrt{\frac{C_{D,0}}{K}}}{2C_{D,o}} = \frac{1}{2}\sqrt{\frac{1}{C_{D,0}K}}. \quad \textbf{(32)}$$

Using this equation, the theoretical $\left(\frac{L}{D}\right)_{max}$ can be calculated in one of two ways.
The first way is by using a calculated value for $C_{D,0}$ through using eq. 23. Values of K, C_L, and e_v are found through eq. 15, 17, and 6 to be 0.0664, 2.47, and 0.852, respectively; these values are then substituted into eq. 23, resulting in $C_{D,0}$ = 0.164. From here, $\left(\frac{L}{D}\right)_{max}$ is calculated through eq. 32, and it is found that $\left(\frac{L}{D}\right)_{max}$ = 4.79.

The second way to calculate $\left(\frac{L}{D}\right)_{max}$ is by using the value of $C_{D,0}$ given by *Reference 3,* which was used previously to predict values for the NACA 6412. This value of $C_{D,0}$ was found to be 0.00799, resulting in $\left(\frac{L}{D}\right)_{max}$ = 21.14. These two values of $C_{D,o}$ are not similar, due to the calculated $C_{D,0}$ being found using a speed of 4.5 m/s, which was the theoretical U for $\left(\frac{L}{D}\right)_{max}$, while the second $C_{D,0}$ was calculated according to the airfoil dimensions for a NACA 6412. The second $C_{D,0}$ was much higher and more accurate to predicted values of $C_{D,0}$ for this type of aircraft.

The experimental value of $\left(\frac{L}{D}\right)_{max}$ can be found by

$$\left(\frac{L}{D}\right)_{\text{max}} = \frac{R}{h},$$

a rewritten version of (Eq. 26). Using the maximum range from the measured data in Table 1, R is calculated to be 29.5m, and the launch height is known to be 1.75m. From these measurements, the experimental value of $\left(\frac{L}{D}\right)_{max}$ = 16.9.

The difference between the theoretical and actual values can be explained by several differences in theory versus launch day, with some notable differences being that the glider weighed 0.85 kg on flight day, which is different from the second theoretical value, which used 0.6 kg, as well as the experimental U for $\left(\frac{L}{D}\right)_{max}$, which was 9.2 m/s instead of the theoretical 4.5 m/s. Due to the large variation in these quantities, the theoretical and experimental values of $\left(\frac{L}{D}\right)_{max}$ vary greatly. As referenced in (S4.1), the theoretical value of $C_{D,o}$ is more than double the actual value of $C_{D,o}$, which accounts for the drastic difference between the first value of $C_{D,o}$, which was calculated based on the theoretical speeds which would yield $\left(\frac{L}{D}\right)_{max}$, while the second $C_{D,o}$ was based on airfoil geometry, and therefore more accurate.

4.4 U_γ Comparison

To predict a value for U_γ, which is the flight speed that minimizes γ, the glide angle, (Eq. 25) (derived in S1.1), which is shown below,

$$U_\gamma^2 = \frac{2W}{\rho S}\sqrt{\frac{K}{C_{D,0}}} \quad \textbf{(25)}$$

can be rearranged to become:

$$U_\gamma = \sqrt{\left(\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D,0}}}\right)} \quad \textbf{(33)}$$

It is shown that U_γ depends on 5 total variables, K, $C_{D,0}$, W, ρ, and S, which all have known values when theoretical conditions for the plane are assumed. Since these variables are all known from previous sections, they will be reiterated for convenience.

$W = 5.88N$

$\rho = 1.23\ kg/m^3$

$$S = 0.269\ m^2$$
$$K = 0.0707$$
$$C_{D,0} = 0.00799$$

By plugging these values into (Eq. 33),

$$U_\gamma = \sqrt{\left(\frac{2W}{\rho S}\sqrt{\frac{K}{C_{D,0}}}\right)} = \sqrt{\left(\frac{2(5.88)}{(1.23)(0.269)}\sqrt{\frac{(0.0707)}{(0.00799)}}\right)} = 10.3\ \text{m/s}$$

it is found that the predicted value for U_γ is 10.3 m/s.

Since U_γ is the flight speed which minimizes γ, it makes sense that U_γ is also the flight speed that achieves the maximum range, R_{max}. Table 2 shows that flight 7, which achieved R_{max}, flew at a speed of 9.2 m/s, therefore the measured value for U_γ is 9.2 m/s. This is lower than the predicted value for U_γ , which was 10.3 m/s.

4.5 C_L at D_{min}

In order to calculate C_L using U_γ eq. 6 can be rewritten, giving the equation

$$C_L = \frac{L}{qS}. \qquad \textbf{(34)}$$

Eq. 34 can be further manipulated by substituting in eq. 5 and eq. 7, giving the equation

$$C_L = \frac{W}{\frac{1}{2}\rho U^2 S}. \qquad \textbf{(35)}$$

By plugging in the predicted U_γ, 10.3 m/s, for flight speed, along with previously measured values of W = 8.33N, ρ = 1.24 kg/m^3, and S = 0.269 m^2,

$$C_L = \frac{8.33}{\frac{1}{2}(1.24)(10.3)^2(0.269)} = 0.471$$

it is found that the value for C_L is 0.471. This value is lower than the value for C_L previously calculated from the glider geometry in HW9, which was 0.754.

V. Summary

The maximum range of the glider was theoretically calculated to occur at a speed of 7.6 m/s which depended on the drag coefficient, mass, and takeoff height. However, experimental results showed variations due to the surface roughness by tape and adjustments made during test flights. The glider was consistent through low and medium speeds. An increase in weight predicted that the glider would reach its maximum range at higher speeds. The experimental data showed that the glider reached the maximum range when the speed was 9.2m/s, evidencing that the actual drag coefficient was lower than initially predicted.

The increased weight of the glider positively impacted its lift, allowing it to maintain reasonable ranges even with a lower takeoff height than theoretical data from previous calculations. Adjustments to the drag coefficient–decreasing from 0.02 to 0.01–created a theoretical performance that was closer to the glider's experimental results, and suggested that total drag played a significant role in determining range.

The wing loading of the glider was consistent with values observed in other lightweight flying devices, as the value of the logarithmic of the glider's wing loading and weight fell within the reasonable range (Fig 16). This supports the justification of the experimental results and demonstrates that the glider was capable of maintaining efficient lift-to-drag ratios for extended flight durations.

The report highlighted the glider's improved performance at higher speeds than predicted, suggesting that the aerodynamic design was more favorable than first calculated. The added weight, while expected to reduce performance, actually positively stabilized the flight path, allowing the glider to achieve longer ranges.

VI. References

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